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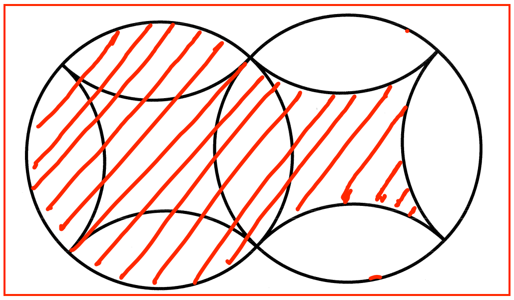
Situation 1: Introduction to Surface Area

1. Prompt



The course was the second semester of a Plane Geometry course; the students had all passed Algebra 1 the previous year. This lesson introduced the unit on surface area and volume, and this problem was the only one the students worked on for thirty-five minutes. (The teacher spent five minutes at the end of class discussing homework.) Each student was given the shape above (without any shading) on card stock and a pair of scissors with which they cut out the curved shape and then scored the interior arcs. They then folded "outward" on the scored arcs and then folded the paper together to form a solid. (Three opposite pairs of the  shape overlap.) Given that the radius of each circle is *r*, the problem is to find the surface area of the solid created. The students were placed in groups and the teacher circulated through the class to monitor, encourage, and check for understanding. The students came up with three different solutions that were discussed at the end of the class (before the teacher went over homework).

Mathematical Focus 1
Students need to know the difference between surface area and volume.

Mathematical Focus 2
The surfaces of the solid in this lesson, like that of many solids familiar to students, can be unfolded and rolled onto a plane so that area in this situation can be calculated in the same way that two-dimensional areas are computed.

Mathematical Focus 3
Decomposition and recomposition of planar regions preserve area.

Mathematical Focus 4
There are well-known formulas for the areas of circles, triangles, and rectangles.

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Situation 2: Surface Area in Calculus

2. Prompt

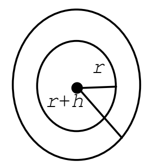
In a BC Calculus that had already completed the unit on volume, the teacher in a lesson on surface area proved that the surface area of a sphere with radius r is given by $S = 4\pi r^2$. A student noticed that this formula was the derivative with respect to r of $V = \frac{4}{3}\pi r^3$. She also remarked that in two dimensions, if A is the area of a circle with radius r, then $\frac{dA}{dr} = 2\pi r$, which is the circumference of the circle. She asked the teacher if these were coincidences.

Mathematical Focus 1

Using the definition of the derivative shows that

$$\frac{dA}{dr} = \lim_{h\to 0}\frac{A(r+h) - A(r)}{h} = \lim_{h\to 0}\frac{\pi(r+h)^2 - \pi r^2}{h} = \lim_{h\to 0}(2\pi r + \pi h) = 2\pi r = C.$$

Mathematical Focus 2



The ring is the area between the circle of radius r and the concentric circle of radius $r + h$. As the width of the ring gets smaller, the relative change in the area of the circle approaches the circumference (as suggested by the numerator of the difference quotient).

Mathematical Focus 3

Analogous to Focus 1, $\frac{dv}{dr} = S$.

Mathematical Focus 4

An argument with spheres similar to Focus 2 will give a geometric interpretation of this relationship.

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